MATHEMATICS

2nd Prep



Collected by:

MR. ESLAM YOUSSIF

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Unit 1

Real Numbers

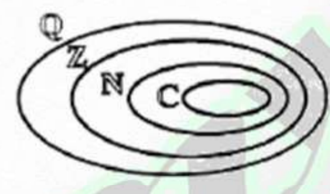
Mr. Eslam Youssif

0122 67 666 55

1-1 Revision

We had studied before the following sets of numbers :

- The set of counting numbers : $\mathbb{C} = \{1, 2, 3, 4, ...\}$
- The set of natural numbers : $\mathbb{N} = \{0, 1, 2, 3, ...\} = \mathbb{C} \cup \{0\}$
- The set of integers : $\mathbb{Z} = \{..., 3, 2, 1, 0, -1, -2, -3, ...\}$
- The set of positive integers : $\mathbb{Z}_+ = \{1, 2, 3, ...\} = \mathbb{C}$
- The set of negative integers : $\mathbb{Z}_- = \{-1, -2, -3, ...\}$
- $\mathbb{Z} = \mathbb{Z}_+ \cup \{0\} \cup \mathbb{Z}_-$
- The set of rational numbers : $\mathbb{Q} = \{ \frac{a}{b} : a \in \mathbb{Z}, b \in \mathbb{Z}, b \neq 0\}$
- $\mathbb{C} \subset \mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q}$



the absolute value

- $|4| = 4$, $|-4| = 4$, $|0| = 0$
- If $|x| = a$, then $x = \pm a$ **For example** : If $|x| = 5$, then $x = \pm 5$

If a and b are two rational numbers , m and n are two integers then :

- $a^{-n} = \frac{1}{a^n}$ **For example** : $5^{-1} = \frac{1}{5}$
- $a^m \times a^n = a^{m+n}$ **For example** : $2^3 \times 2^2 = 2^{3+2} = 2^5 = 32$

- $\frac{a^m}{a^n} = a^{m-n}$ For example : $\frac{3^2}{3^{-1}} = 3^{2-(-1)} = 3^{2+1} = 3^3 = 27$
- $(ab)^n = a^n b^n$ For example : $(5 \times 10)^2 = 5^2 \times (10)^2 = 25 \times 100 = 2500$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ For example : $\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$
- $(a^m)^n = a^{mn}$ For example : $(2^2)^3 = 2^{2 \times 3} = 2^6 = 64$

The square root of the perfect square rational number (a)

- $\sqrt{16} = 4$
- $-\sqrt{16} = -4$
- $\pm\sqrt{16} = \pm 4$
- $\sqrt{0} = 0$
- $\sqrt{\text{negative rational number}}$ is meaningless in $\mathbb{Q}$
- $\sqrt{a^2} = |a|$

Find the solution set of each of the following equations :

1) $x + 2 = |-2| , x \in \mathbb{N}$

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2) $2x - 5 = 13 , x \in \mathbb{Q}$

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3) $x^2 - 4 = 5 , x \in \mathbb{Q}$

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4) $x^2 + 25 = 0 , x \in \mathbb{Q}$

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1-2 The cube root of a rational number

The cube root of the rational number "a" is the number whose cube equal to a

- The cube root of the rational number "a" is denoted by $\sqrt[3]{a}$
- The cube root of any number has the same sign of this number.
- $\sqrt[3]{a^n} = a^{\frac{n}{3}}$ where $n \in \mathbb{Z}$
- The cube root of a perfect cube rational number is also a rational number.

Find each of the following :

1) $\sqrt[3]{216}$

2) $\sqrt[3]{\frac{-8}{125}}$

3) $\sqrt[3]{0.064}$

4) $\sqrt[3]{1728}$

5) $-\sqrt[3]{0.216}$

6) $\sqrt[3]{-3\frac{3}{8}}$

Solve each of the following equations in $\mathbb{Q}$:

7) $40x^3 - 1 = -136$

8) $(y-2)^3 = -343$

9) $27x^3 - 2 = 62$

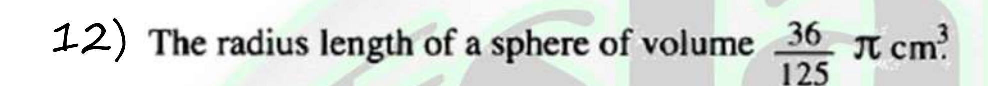
10) $(5x-3)^3 - 2 = 6$

Notice that :

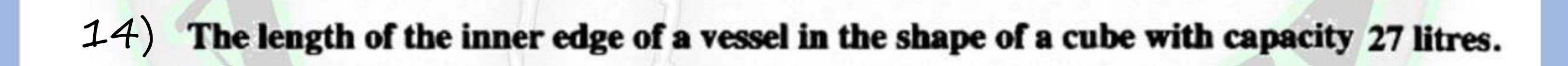
- The area of one face of a cube = the edge length × itself
- The lateral area of a cube = the area of one face × 4
- The total area of a cube = the area of one face × 6
- The volume of the sphere = $\frac{4}{3}\pi r^3$
- 1 litre = 1000 cm^3

Find :

11) The length of the inner edge of a vessel in the shape of a cube if its capacity = 8 litres.


12) The radius length of a sphere of volume $\frac{36}{125}\pi$ cm^3.

13) The diameter length of a sphere of volume equals 38808 cm^3. $(\pi \approx \frac{22}{7})$


14) **The length of the inner edge of a vessel in the shape of a cube with capacity 27 litres.**

15) **The length of the diameter of a sphere of volume 36 π cm^3.**

1-3 The set of irrational numbers

- We studied that the number would be a rational number If it can be written in the form $\frac{a}{b}$ where $a \in \mathbb{Z}$, $b \in \mathbb{Z}$ and $b \neq 0$
- **The square root of the perfect square rational number is a rational number.**
- **The cube root of the perfect cube rational number is a rational number.**

There is another set of numbers which are not rational numbers. This set is called "the set of irrational numbers" and it is denoted by $\grave{\mathbb{Q}}$

- $\mathbb{Q}$ and $\grave{\mathbb{Q}}$ are disjoint sets.
- $\mathbb{Q} \cap \grave{\mathbb{Q}} = \varnothing$

Show which of the following numbers belongs to $\mathbb{Q}$ and which of them belongs to $\grave{\mathbb{Q}}$:

1) $\sqrt{0.49}$
2) $\sqrt{\frac{25}{49}}$
3) $\sqrt{25} + \sqrt[3]{16}$
4) $\sqrt[3]{-0.064}$
5) $\sqrt[3]{\frac{25}{49}}$
6) 3
7) $\sqrt{9}$
8) 5
9) $\sqrt{3}$
10) -8
11) $\sqrt[3]{5}$
12) $\sqrt[3]{-8}$

If $x \in \grave{\mathbb{Q}}$ find the S.S. of each of the following equations :

13) $x^2 = 5$

14) $\frac{2}{5} x^2 = \frac{4}{25}$

15) $(x^2 - 10)(x^3 - 4) = 0$

....

....

....

16) $x^3 = 7$

17) $64x^3 - 2 = -29$

18) $\frac{1}{2}x^2 - 5 = 3$

Remark

- Each irrational number lies between two rational numbers.
- Each irrational number can be represented by a point on the number line.

Prove that :

19) $\sqrt{3}$ lies between 1.7 and 1.8

20) $\sqrt[3]{12}$ lies between 2.2 and 2.3

21) $\sqrt{7}$ lies between 2.6 and 2.7

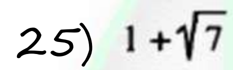
Find two consecutive integers such that

22) $\sqrt{13}$ **lies between them.**

Determine the point which represents the number on the number line.

23) $\sqrt{7}$

24) $-\sqrt{7}$



25) $1+\sqrt{7}$

26) $2-\sqrt{7}$

27) $2\sqrt{7}$

28) $\sqrt{5}$

29) **Find the length of the diagonal of a square whose area = 5 cm^2**

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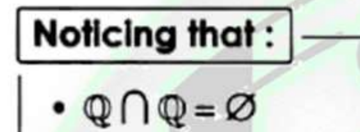
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1-4 The set of real numbers

The set of real numbers

It is the set obtained from the union of the set of rational numbers and the set of irrational numbers. It is denoted by $\mathbb{R}$

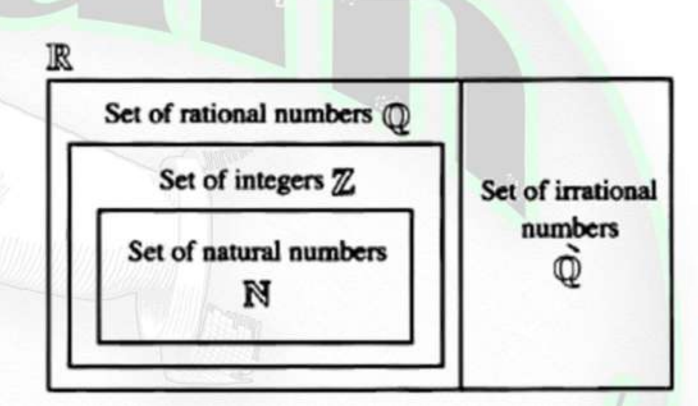
i.e. $\mathbb{R} = \mathbb{Q} \cup \grave{\mathbb{Q}}$ (as shown in the opposite figure)



- **The opposite Venn diagram shows that :**

  $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

  and $\grave{\mathbb{Q}} \subset \mathbb{R}$



Arrange the following numbers ascendingly :

1) $\sqrt{75}$, $\sqrt{68}$, $-\sqrt{45}$, -8 , 7 and $-\sqrt{32}$

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....

Complete each of the following using the suitable symbols > or < :

2) $\sqrt{2}$ 1

3) $-\sqrt[3]{7}$ .... -2

4) $-\sqrt{3}$ .... -1

5) $\sqrt{7}$ 2.6

6) $\sqrt[3]{9}$ 3

7) $-\sqrt[3]{16}$ .... -2.52

Write three irrational numbers included between the two numbers

8) **11 and 12**

Find the S.S. in $\mathbb{R}$ for each of the following equations :

9) $3x^2 + 125 = 221$

10) $\frac{1}{6}x^3 - 8 = 28$

11) $2x^2 + 6 = 4$

1-5 Intervals

First : The limited intervals :

- **A** The closed interval :

- **B** The opened interval :

- **C** The half opened interval (the half closed interval) :

Second : The unlimited intervals :

Remarks

- $\mathbb{R} =]-\infty , \infty[$
- $\mathbb{R}_+ =]0 , \infty[$
- $\mathbb{R}_- =]-\infty , 0[$
- The set of non-negative real numbers $= \mathbb{R}_+ \cup \{0\} = [0 , \infty[$
- The set of non-positive real numbers $= \mathbb{R}_- \cup \{0\} =]-\infty , 0]$

Write each of the following sets in the form of an interval , then represent it on the number line.

1) $\{x : x \in \mathbb{R}, -3 < x \le 0\}$

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2) $\{x : x \in \mathbb{R}, x > 0\}$

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3) $\{a : a \in \mathbb{R}, 1 \ge a \ge -2\}$

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4) $\{y : y \in \mathbb{R}, -1 \ge y\}$

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5) $\{x : x \in \mathbb{R}, -4 < x \le 2\}$

....

....

6) $\{y : y \in \mathbb{R}, y \ge -5\}$

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Represent each of the following on the number line and express it by the description method :

7) $]-3, 0[$

....

....

8) $]-\infty, 2]$

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Operations on intervals

- $X \cap Y$ = the set of elements which are common in X and Y
- $X \cup Y$ = the set of all elements in X or Y without repeating
- $X - Y$ = the set of elements which are in X and not in Y
- $Y - X$ = the set of elements which are in Y and not in X
- $\grave{X} = U - X$

If X = [– 3 , 3] and Y = [– 1 , 5[, find using the number line :

9) X ∪ Y ....

10) X ∩ Y ....

11) X – Y ....

12) Y – X ....

If X =] – ∞ , 2 [and Y = [– 1 , 5] , find using the number line :

13) X ∪ Y ....

14) X ∩ Y ....

15) X – Y ....

16) Y – X ....

17) X̀ ....

18) Ỳ ....

If X = [1 , 4 [and Y = {1 , 4} , find :

19) X ∩ Y ....

20) X ∪ Y ....

21) X – Y ....

22) Y – X ....

If X = [– 1 , 3 [and Y =]0 , 4] , find using the number line :

23) $X \cap Y$

24) $X \cup Y$

25) $X - [0 , \infty[$

26) $]0 , \infty[- Y$

27) $X`$

28) $X \cap \{-2 , -1 , 0 , 1 , 2 , 3\}$

Find each of the following :

29) $]-\infty , 2] \cap]-3 , \infty[$

30) $[5 , \infty[-]5 , \infty[$

31) $]-\infty , 3] \cup [-2 , 5[$

32) $[2 , \infty[\cap]-\infty , 2[$

1-6 Operations on the real numbers

The properties of addition operation of real numbers

- **Closure :**

  For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ we find that $(a + b) \in \mathbb{R}$

- **Commutative property :**

  For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a + b = b + a$

- **Associative property :**

  For every $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $c \in \mathbb{R}$ it will be $(a + b) + c = a + (b + c) = a + b + c$

- **The additive neutral :**

  For every $a \in \mathbb{R}$ it will be $a + 0 = 0 + a = a$

- **The additive inverse of every real number :**

  For every $a \in \mathbb{R}$ there is $(-a) \in \mathbb{R}$ where $a + (-a) =$ zero (the additive neutral)

- Subtraction operation is not commutative and it is not associative.

Write the additive inverse for every of the following numbers :

1) $\sqrt{2}$ ....

2) $-\sqrt[3]{5}$ ....

3) $\sqrt{2}+\sqrt{7}$ ....

4) $\sqrt[3]{5}-3$ ....

5) $-\sqrt{6}-\sqrt[3]{7}$ ....

Simplify to the simplest form :

6) $-3\sqrt{5}+4\sqrt{5}+(-2\sqrt{5})$

7) $4+\sqrt{3}-7-\sqrt{3}$

8) $2+2\sqrt{7}-1-5\sqrt{7}$

9) $3\sqrt{5}+\sqrt{3}-3\sqrt{5}+5\sqrt{3}$

The properties of multiplication operation of real numbers

- **Closure :**
  For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a \times b \in \mathbb{R}$
- **Commutative property :**
  For every $a \in \mathbb{R}$ and $b \in \mathbb{R}$ it will be $a \times b = b \times a$
- **The associative property :**
  For every $a \in \mathbb{R}$, $b \in \mathbb{R}$ and $c \in \mathbb{R}$ it will be $(a \times b) \times c = a \times (b \times c) = a \times b \times c$
- **The multiplicative neutral :**
  For every $a \in \mathbb{R}$ it will be $a \times 1 = 1 \times a = a$
- **The multiplicative inverse of any non-zero real number :**
  For every real number $a \neq 0$, there is a real number $\frac{1}{a}$ where $a \times \frac{1}{a} = 1$
- The division operation in $\mathbb{R}$ is not commutative and it is not associative.
- **Distributing multiplication on addition and subtraction**

$a(b \pm c) = ab \pm ac$

Find the result of each of the following :

10) $-2 \times 3\sqrt{5}$

11) $4\sqrt{2} \times \sqrt{2}$

12) $-2\sqrt{7} \times 4\sqrt{7}$

13) $\frac{\sqrt{5}}{5} \times \frac{4\sqrt{5}}{12\sqrt{2}} \div \frac{1}{3\sqrt{2}}$

14) $\sqrt{5} \times \frac{1}{\sqrt{5}} \times \sqrt{5}$

15) $\frac{\sqrt{3}}{3} \times \frac{4\sqrt{5}}{20} \times \frac{5\sqrt{3}}{\sqrt{5}}$

Write each of the following such that the denominator is an integer :

16) $\frac{9}{\sqrt{3}}$

17) $-\frac{3}{\sqrt{2}}$

18) $\frac{5}{3\sqrt{5}}$

19) $\frac{3}{\sqrt{7}}$

20) $\frac{9}{2\sqrt{6}}$

Find each of the following :

21) $2\sqrt{3}(5\sqrt{3}-4)$

22) $5\sqrt{2}(3\sqrt{2}-2)$

23) $(7\sqrt{2}-5)(7\sqrt{2}+5)$

24) $(2\sqrt{3}-3)(2\sqrt{3}+3)$

25) $(2+\sqrt{3})(\sqrt{3}+7)$

26) $(5\sqrt{3}-2)^2$

Find the value of the expression :

27) $x^2+2xy+y^2$, If $x=5\sqrt{3}-2$, $y=5\sqrt{3}+2$

28) $x^2-2xy+y^2$, If $x=2\sqrt{3}-1$ and $y=2\sqrt{3}+1$

1-7 Operations on the square roots

- $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$
- $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$ where $b \neq 0$
- $\frac{\sqrt{a}}{\sqrt{b}} = \frac{\sqrt{a}}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}} = \frac{\sqrt{ab}}{b}$ where $b \neq 0$
- $a\sqrt{b} = \sqrt{a^2 b}$
- $\sqrt{a^2 + b^2} \neq a + b$
- $\sqrt{a^2 - b^2} \neq a - b$

Write each of the following in the form $a\sqrt{b}$ where a and b are two integers , b is the least possible value.

1) $\sqrt{27}$

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2) $5\sqrt{54}$

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3) $3\sqrt{\frac{2}{3}}$

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4) $\frac{\sqrt{84}}{\sqrt{7}}$

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Simplify to the simplest form :

5) $\sqrt{45} - 2\sqrt{20} + 2\sqrt{5}$

6) $2\sqrt{18} + \sqrt{50} - 42\sqrt{\frac{1}{2}}$

7) $2\sqrt{27} - 3\sqrt{\frac{1}{3}} - \frac{6}{\sqrt{3}}$

8) $\sqrt{75} - 2\sqrt{27} + \sqrt{3}$

9) $2\sqrt{50} - 3\sqrt{2} - 4\sqrt{\frac{1}{8}}$

Find each of the following :

10) $2\sqrt{3}(\sqrt{6} + 5)$

11) $(3\sqrt{2} - 5)(3\sqrt{2} + 5)$

12) $(\sqrt{2}+\sqrt{6})^2$

....

....

....

find the value of

13) $a^2 + 2\sqrt{3}$, If $a = \frac{\sqrt{6}-\sqrt{2}}{\sqrt{2}}$

....

....

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Write each of the following such that the denominator is an integer :

14) $\frac{5\sqrt{3}}{2\sqrt{5}}$

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15) $\frac{1+\sqrt{3}}{3\sqrt{3}}$

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1-8 the two conjugate numbers

- $(\sqrt{a} + \sqrt{b})$ and $(\sqrt{a} - \sqrt{b})$ is conjugate to the other
- Their sum = $(\sqrt{a} + \sqrt{b}) + (\sqrt{a} - \sqrt{b}) = 2\sqrt{a}$ = twice the first term.
- Their product = $(\sqrt{a} + \sqrt{b})\ (\sqrt{a} - \sqrt{b}) = (\sqrt{a})^2 - (\sqrt{b})^2 = a - b$
- The product of the two conjugate numbers is always a rational number.

Important Remarks

- $(x - y)(x + y) = x^2 - y^2$
- $(x + y)^2 = x^2 + 2xy + y^2$
- $x^2 + xy + y^2 = (x + y)^2 - xy$
- $x^2 + y^2 = (x + y)^2 - 2xy$
- $(x - y)^2 = x^2 - 2xy + y^2$
- $x^2 - xy + y^2 = (x - y)^2 + xy$
- $x^2 + y^2 = (x - y)^2 + 2xy$

Complete the following :

1) $(\sqrt{2} + \sqrt{5})$ its conjugate is .... , their sum = ... , their product = ...

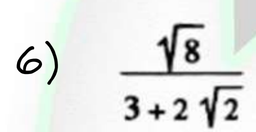
2) $(3 - \sqrt{7})$ its conjugate is .... , their sum = ... , their product = ...

3) $(3\sqrt{5} - \sqrt{6})$ its conjugate is .... , their sum = ... , their product = ...

Write each of the following such that the denominator is a rational number.

4) $\frac{4}{\sqrt{7}-\sqrt{3}}$

5) $\frac{12}{\sqrt{6}-\sqrt{2}}$

6) 
$\frac{\sqrt{8}}{3+2\sqrt{2}}$

7) If $x = \frac{4}{2-\sqrt{2}}$ and $y = \frac{3-2\sqrt{2}}{3+2\sqrt{2}}$,

write each of x and y such that its denominator is a rational number , then find $x + y$

8) If $x = \frac{2}{\sqrt{5}-\sqrt{3}}$ and $y = \sqrt{5}-\sqrt{3}$,

prove that x and y are conjugate numbers , then find the value of the expression :

a. $x^2 + 2xy + y^2$

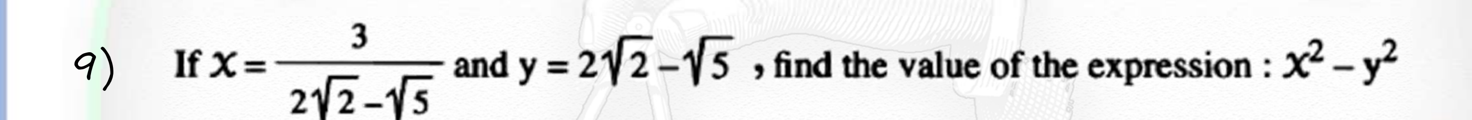
b. $x^2 + xy + y^2$

....

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9) If $x = \frac{3}{2\sqrt{2}-\sqrt{5}}$ and $y = 2\sqrt{2}-\sqrt{5}$, find the value of the expression : $x^2 - y^2$

....

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1-9 Operations on the cube roots

If a and b are two real numbers , then

- $\sqrt[3]{a} \times \sqrt[3]{b} = \sqrt[3]{ab}$
- $\dfrac{\sqrt[3]{a}}{\sqrt[3]{b}} = \sqrt[3]{\dfrac{a}{b}}$ (where $b \neq 0$)
- $\sqrt[3]{a^3 + b^3} \neq a + b$
- $\sqrt[3]{a^3 - b^3} \neq a - b$
- $\sqrt[3]{-a} = -\sqrt[3]{a}$
- $a\sqrt[3]{b} = \sqrt[3]{a^3 b}$
- $\sqrt[3]{\dfrac{a}{b}} = \sqrt[3]{\dfrac{a}{b} \times \dfrac{b^2}{b^2}} = \sqrt[3]{\dfrac{a\,b^2}{b^3}} = \dfrac{1}{b}\sqrt[3]{a\,b^2}$

Simplify each of the following in the simplest form :

1) $\sqrt[3]{\dfrac{2}{3}} \times \sqrt[3]{\dfrac{4}{9}}$

....
....
....
....

2) $\sqrt[3]{\dfrac{5}{4}} \div \sqrt[3]{\dfrac{2}{25}}$

....
....
....
....

3) $\sqrt[3]{24}+\sqrt[3]{3}-\sqrt[3]{81}$

4) $\sqrt[3]{54}+6\sqrt[3]{16}-6\sqrt[3]{\frac{1}{4}}$

5) $\sqrt[3]{81}+\sqrt{12}-2\sqrt[3]{3}-2\sqrt{3}$

6) $2\sqrt[3]{4}\left(5\sqrt[3]{\frac{1}{2}}-\sqrt[3]{32}\right)$

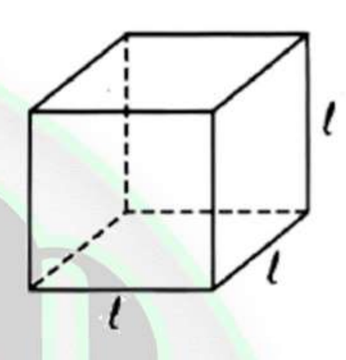
7) $5\sqrt[3]{2}-\sqrt{16}+\sqrt[3]{-54}$

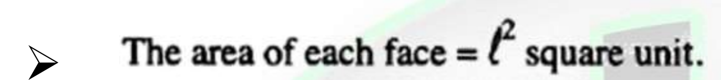
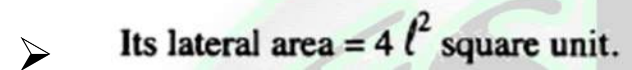
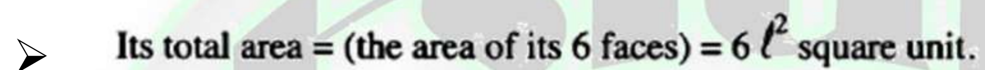
8) $\sqrt[3]{72}+\sqrt[3]{\frac{1}{3}}+\sqrt[3]{-9}$

9) If $x=\sqrt[3]{5}+2$ and $y=\sqrt[3]{5}-2$, find the value of $(x+y)^3-(x-y)^3$

1-10 Applications on the real numbers

The Cube



-  The area of each face = l^2 square unit.
-  Its lateral area = $4\,l^2$ square unit.
-  Its total area = (the area of its 6 faces) = $6\,l^2$ square unit.
- Its volume = l^3 cube unit.

find

1) **A cube with volume 125 cm.^{3} , find its total area and its lateral area.**

....

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....

....

....

Complete the following table

2)

	Edge length of the cube	Area of one face	lateral area	Total area	Volume
(1)	3 cm.	…………………	…………………	…………………	…………………
(2)	…………………	49 cm^2.	…………………	…………………	…………………
(3)	…………………	…………………	144 cm^2.	…………………	…………………
(4)	…………………	…………………	…………………	150 cm^2.	…………………
(5)	…………………	…………………	…………………	…………………	64 cm^3.

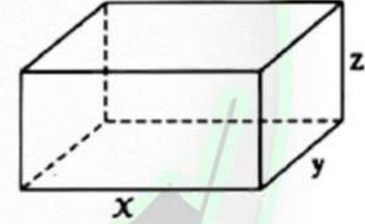
The Cuboid

- Its lateral area = the perimeter of the base × height = $2 (X + y) \times z$ square units.
- Its total area (the area of its six faces)



= the lateral area + twice the area of the base

= $2 (X + y) \times z + 2 X y = 2 (X y + y z + z X)$ square units.



- Its volume = the area of the base × the height = $X \times y \times z$ cube unit.

3) **The height of a cuboid is 4 cm. and its base is a square of side length 5 cm. Find**

a. its volume b. its lateral area c. its total area.

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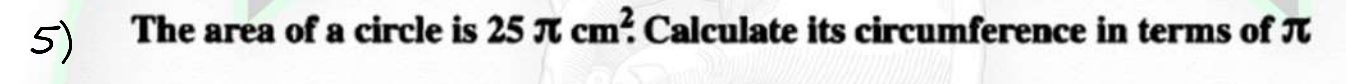
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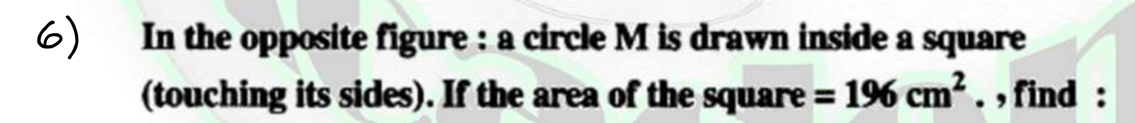
4) The dimensions of a cuboid are 3 cm. , 4 cm. and 5 cm. Calculate its volume and its total area.

The Circle

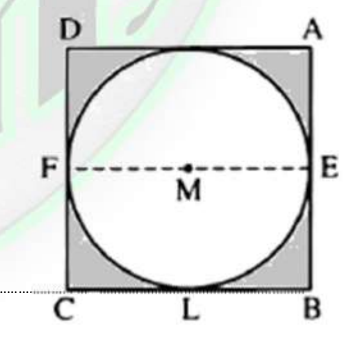
- The circumference of the circle = $2\pi r$ length unit.
- The area of the circle = πr^2 square unit.

5) **The area of a circle is 25π cm^2. Calculate its circumference in terms of π**





6) **In the opposite figure : a circle M is drawn inside a square (touching its sides). If the area of the square = 196 cm^2 . , find :**



a. The area of the shaded part.

b. The perimeter of the shaded part.

....

....

....

7) The circumference of a circle is 88 cm. Find its area.

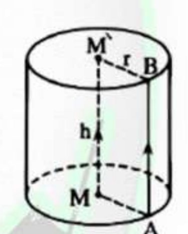
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The right circular cylinder



- The lateral area of the cylinder = 2 π r h square unit.
- The total area of the cylinder = the lateral area of the cylinder + twice the area of the base
  = 2 π r h + 2 π r^2 square unit.
- The volume of the cylinder = the area of the base × height = π r^2 h cube unit.

8) **A right circular cylinder is of height 10 cm. and its volume is 1540 cm^3.**
Find its total area $(\pi = \frac{22}{7})$

....

....

....

....

9) A right circular cylinder is of volume 90π cm.^{3} and its height is 10 cm. Find the diameter length of its base.

....

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....

The Sphere





- The area of the sphere = $4\pi r^2$ square unit.
- The volume of the sphere = $\frac{4}{3}\pi r^3$ cube unit.

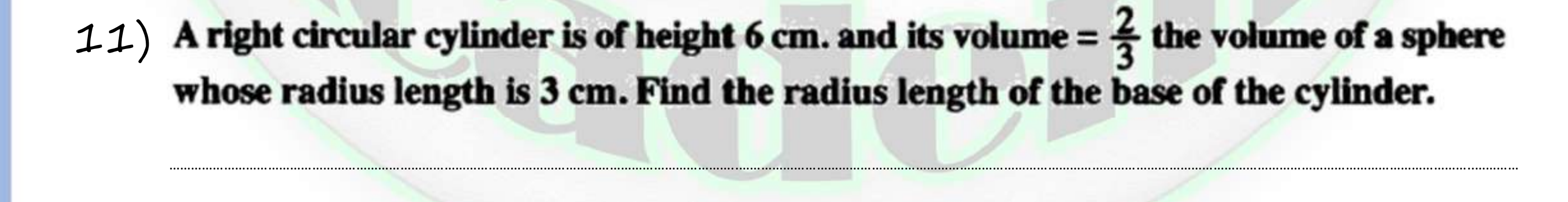
10) **The volume of a sphere = $\frac{500}{3}\pi$ cm.^{3} Find the length of its diameter.**

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11) **A right circular cylinder is of height 6 cm. and its volume = $\frac{2}{3}$ the volume of a sphere whose radius length is 3 cm. Find the radius length of the base of the cylinder.**

....

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12) The area of a sphere is 36π cm^2. Find its volume in terms of π

....

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....

....

The solid		The lateral area	Total area	The volume
The cube		$4\ell^2$	$6\ell^2$	ℓ^3
The cuboid		$2(x+y)\times z$	$2(xy+yz+zx)$	xyz
The cylinder		$2\pi rh$	$2\pi rh+2\pi r^2 = 2\pi r(h+r)$	$\pi r^2 h$
The sphere			$4\pi r^2$	$\frac{4}{3}\pi r^3$

1-11 Solving equations and inequalities of the 1st degree

Solving equations of the first degree in $\mathbb{R}$

Find in $\mathbb{R}$ the S.S of each of the following equations , then represent the solution on the number line.

1) $3x + 2 = 1$

....
....
....
....

2) $7x - \sqrt{7} = 6\sqrt{7}$

....
....
....
....

3) $\sqrt{3}x - 1 = 2$

....
....
....
....

4) $x - \sqrt{5} = 1$

....
....
....
....

5) $2x + 5 = 4$

....
....
....

6) $\sqrt{5}x - 1 = 4$

....
....
....

7) $x - \sqrt{3} = 2$

....
....
....

Solving the inequalities of the first degree in one unknown in $\mathbb{R}$

Find in $\mathbb{R}$ the S.S. of each of the following inequalities , then represent the solution on the number line.

8) $2x + 6 < 2$

9) $5 - 4x \leq -3$

10) $3x - 1 > 8$

11) $2 - 2x > -6$

12) $-3 < 2x - 1 \leq 5$

13) $3 < 3 - 5x < 13$

14) $-16 < 5x + 4 \leq 9$

15) $x - 2 \geq 3x - 5$

16) $x - 1 < 3x - 3 \leq x + 5$

17) $2x + 1 > 4x - 3 > 2x - 11$

Unit 2

Relation between two variables

Mr. Eslam Youssif

0122 67 666 55

2-1 Relation between two variables

1) **What is the different possibilities for a person to pay L.E. 45 using two kinds of bills (banknotes) of L.E. 5 and L.E. 10**

....

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....

2) **Find the different possibilities for a person to pay L.E. 65 of bills (banknotes) of L.E. 5 and L.E. 20**

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Find three ordered pairs satisfying each of the following relations :

3) $3x + y = 5$

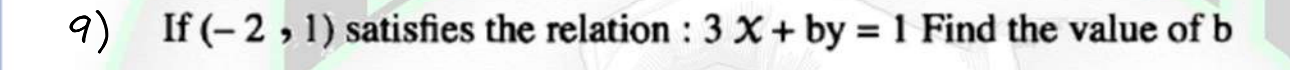
4) $3x - 2y = 6$

5) $3x + y = 2$

6) $2x = 3$

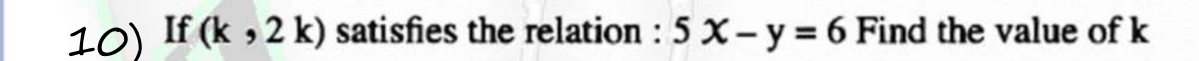
7) $y = -2$

Find :

8) 
Show which of the following ordered pairs satisfies the relation $2x - y = 1$:

$(0, 1), (5, 3), (3, 5), (-2, -5)$

9) If $(-2, 1)$ satisfies the relation : $3x + by = 1$ Find the value of b

10) 
If $(k, 2k)$ satisfies the relation : $5x - y = 6$ Find the value of k

11) **If $(3k, 2k)$ satisfies the relation : $x - 3y = 9$, find the value of k**

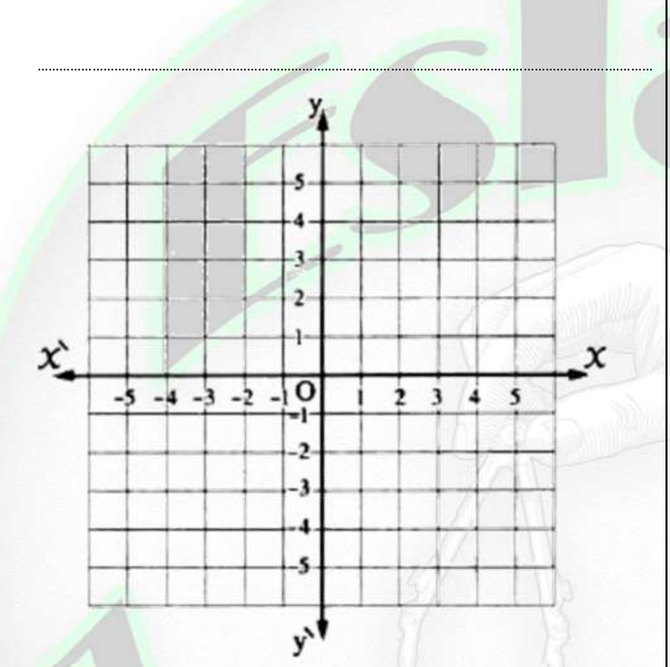
Represent the relation graphically :

12) $2x - y = 3$

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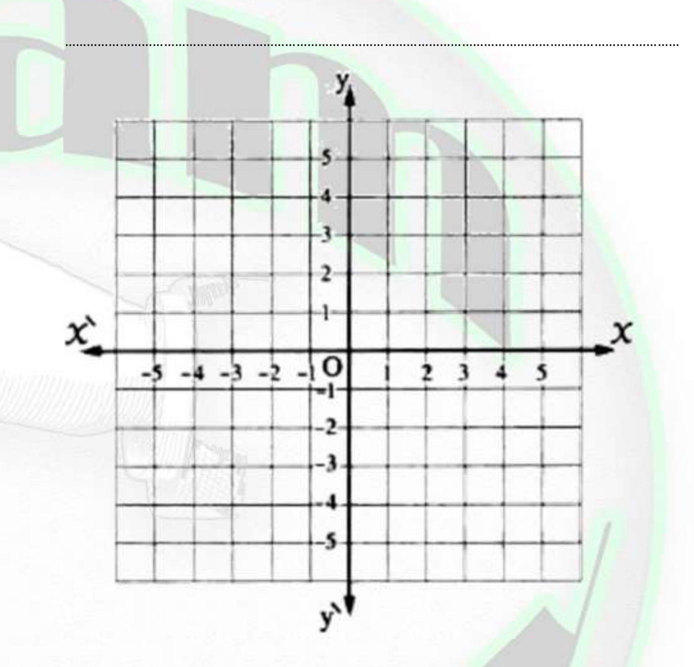


13) $y - 2x = -1$

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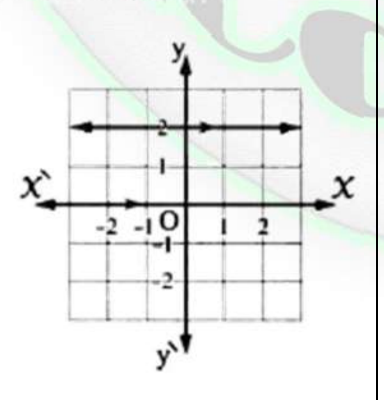
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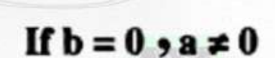
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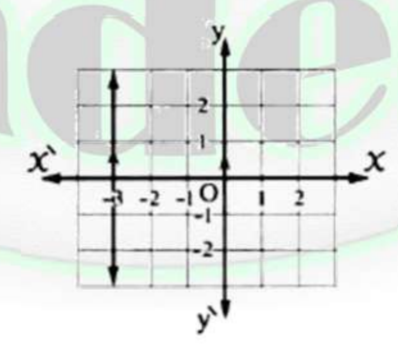


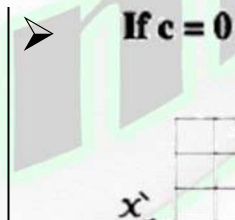
Special cases : a linear relation $ax + by = c$

➢ **If $a = 0$, $b \neq 0$**

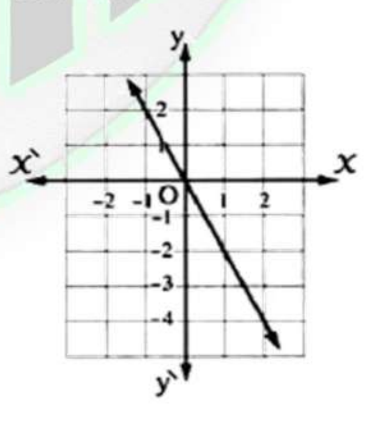


➢ 
If $b = 0$, $a \neq 0$





➢ **If $c = 0$**



14) Graph the straight line which represents the relation : $2x + 5y = 10$ and if this straight line intersects x-axis at the point A and y-axis at the point B , find the area of Δ OAB where O is the origin point.

y
x` x
-5 -4 -3 -2 -1 O 1 2 3 4 5
5 4 3 2 1 -1 -2 -3 -4 -5
y`

2-2 Slope of straight line

- The slope of the straight line = $\frac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}} = \frac{\text{the vertical change}}{\text{the horizontal change}}$
- $S = \frac{y_2 - y_1}{x_2 - x_1}$

Find the slope of the straight line passing through each pair of points in the following :

1) A = (1 , 2) , B = (3 , 3)

2) (2 , 4) , (4 , 5)

3) (– 2 , – 3) , (– 4 , 1)

4) (1 , 3) , (4 , 2)

5) (3 , 1) , (– 1 , 0)

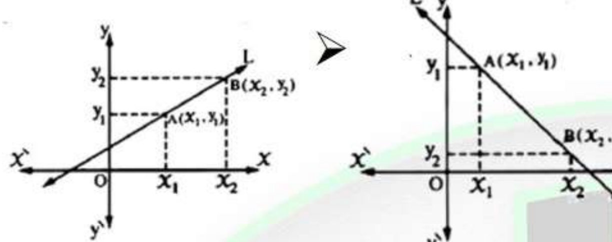
6) (2 , 1) , (3 , 4)

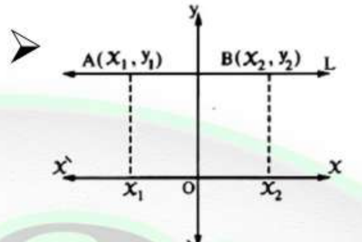
7) (3 , – 5) , (– 4 , 2)

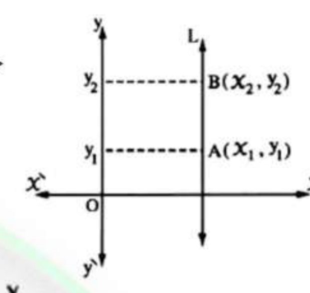
8) (– 3 , – 1) , (1 , 0)

9) (– 6 , 3) , (– 4 , 2)

Remarks



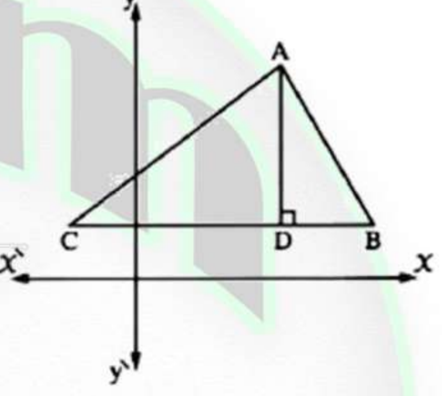




10) **In the opposite figure :**

ABC is a triangle in which $\overline{BC} \parallel \overleftrightarrow{XX'}$, $\overline{AD} \perp \overline{BC}$

Complete the following by choosing one of the words (positive , negative , zero , undefined) in the spaces :



a. The slope of $\overleftrightarrow{AB}$ is ……….

b. The slope of $\overleftrightarrow{AC}$ is ……….

c. The slope of $\overleftrightarrow{BC}$ is ……….

d. The slope of $\overleftrightarrow{AD}$ is ……….

11) **If the slope of the straight line passing through the two points (– 3 , 4) and (1 , y) is 2 , find the value of y**



12) If the slope of the straight line passing through the two points (3 , – 1) , (7 , a) is $\frac{3}{4}$, **find the value of a :**

13) **Prove that :** The points : A (2 , 3) , B (4 , 2) and C (8 , 0) are collinear.

14) **Prove that :** C (– 1 , 2) $\in \overleftrightarrow{AB}$, where A (1 , 3) and B (3 , 4)

15) **If the points : A , B and C are collinear where : A (3 , 2) , B (5 , – 1) and C (1 , k) , find the value of k**

2-3 Real life applications on the slope

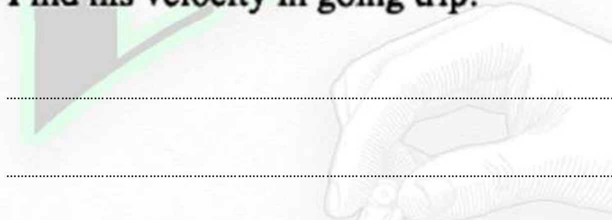
➢ The slope of the straight line which represents this relation = $\dfrac{\text{the change in y-coordinates}}{\text{the change in } x\text{-coordinates}}$

1) **Waleed rode his bicycle from Cairo to Benha , then he returned back to Cairo. The opposite graph represents the bicycle motion during going and returning back :**

Distance in km.
d
50 40 30 20 10
B C
A D
0 2 4 6 8 10
Time in hours
t

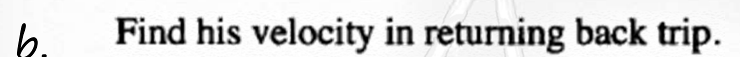


a. Find his velocity in going trip.



....

....



b. Find his velocity in returning back trip.



....

....

c. Find the average velocity during all trips.

....

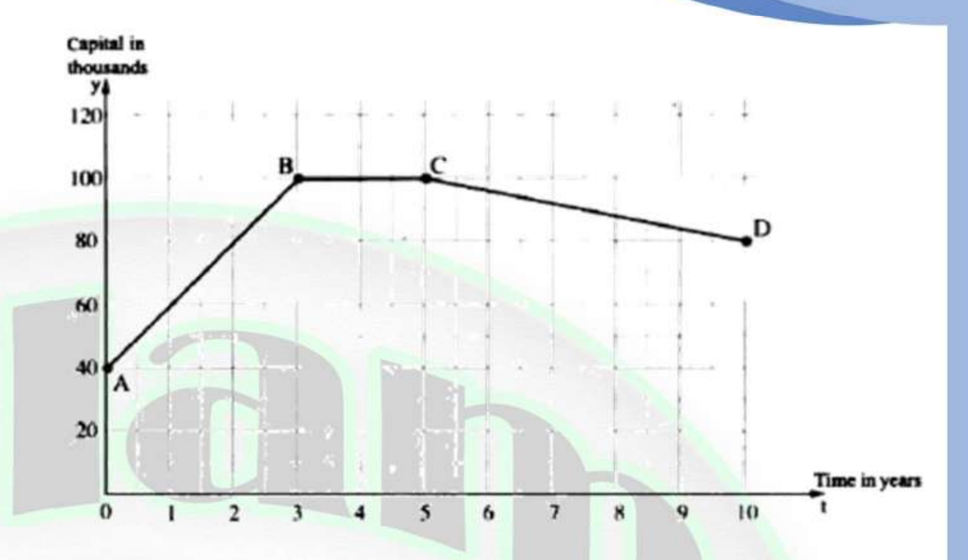
....

d. What do you say about the horizontal line segment in the graph ?

....

....

2) **The opposite graph shows the change of the capital of a company within 10 years :**



a. Find the slope of each of $\overleftrightarrow{AB}$, $\overleftrightarrow{BC}$ and $\overleftrightarrow{CD}$ What is the meaning of each of them ?

....

....

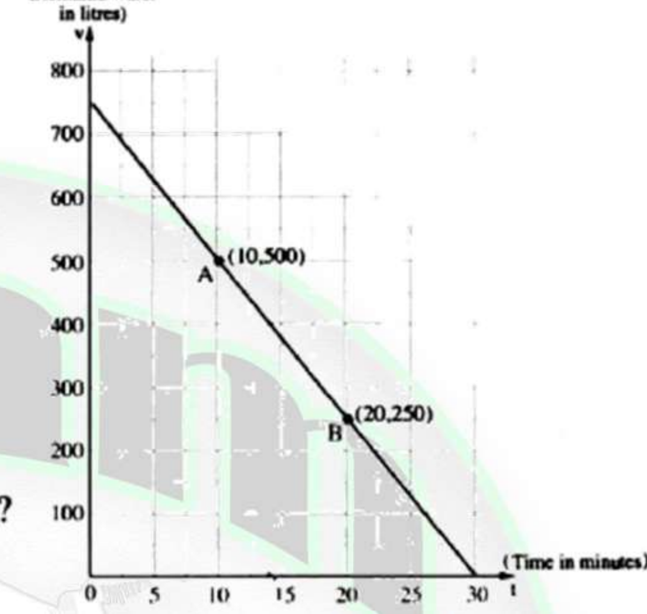
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....

b. Calculate the capital of the company at the beginning.

....

3) A tank of water is filled with water completely. A tap is opened below the tank to empty it , the opposite graph represents the relation between the time (t) in minutes and the amount of water remained in the tank (v) in litres :



a. What is the greatest capacity of the tank ?

....

b. What is the time needed to empty the tank?

....

c. What is the amount remained in the tank after 20 minutes ?

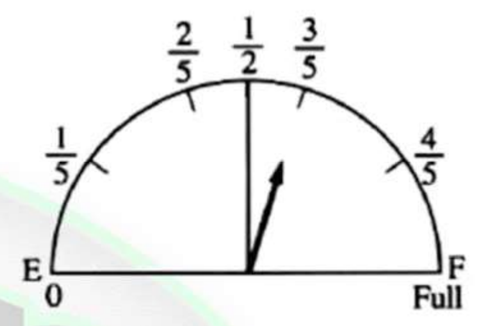
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d. What is the rate of emptying the tank ?

....

....

4) Hossam filled the tank of his car with fuel given that its capacity is 50 litres.
After Hossam covered a distance = 200 km. ,
he noticed that fuel meter shows that the tank has fuel = $\frac{3}{5}$ its capacity.
Graph the relation between the distance covered by the car and the amount of fuel in the tank and calculate the distance covered by the car till the tank becomes empty.



....

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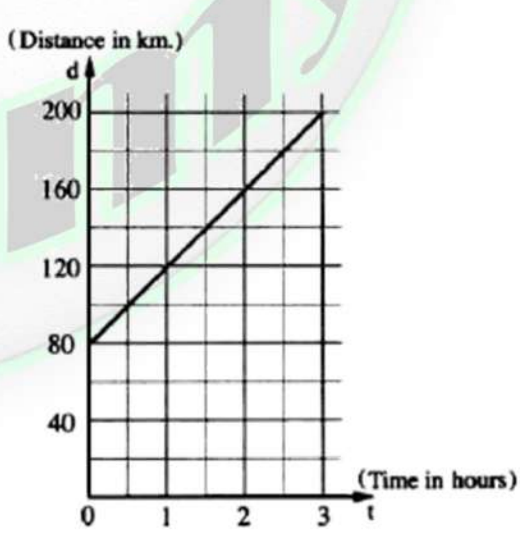
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5) **The opposite graph represents the motion of a car measured from a fixed point A :**



a. Determine the uniform velocity of the car.

....

b. Calculate the covered distance after two hours

....

Unit 3

Statistics



3-1 Collecting and organizing data

1) In the following table , these are the marks of 54 students in one of the classes in grade one preparatory in a school , which they took in an exam in mathematics where the full mark is 60

42	54	36	46	34	45	51	40	48
48	40	47	25	48	45	36	56	44
38	47	30	37.5	40	20	42	28	50
47	55	27	45	30	42	51	43	46
29	43	59	35	44.5	32	24	39	54
41	36	45	39	42	58	35	50	45

The required is forming the frequency table with sets.

Sets	Tallies	Frequency
20 –		
24 –		
28 –		
32 –		
36 –		
40 –		
44 –		
48 –		
52 –		
56 –		
The total		

Sets	Frequency
Total	50

Frequency table with sets

2) **The following is the weights of 50 persons :**

52	35	40	57	43	40	36	49	43	58
47	48	51	30	59	36	45	41	44	37
42	54	38	55	42	47	46	34	53	44
47	32	41	62	50	39	58	46	43	49
40	41	64	44	54	45	38	40	48	41

Form the frequency table with sets using the following two tables :

Sets	Tallies	Frequency
30 –		
35 –		
40 –		
45 –		
50 –		
55 –		
60 –		
Total		50

Tallies table

Sets	Frequency
Total	50

Frequency table with sets

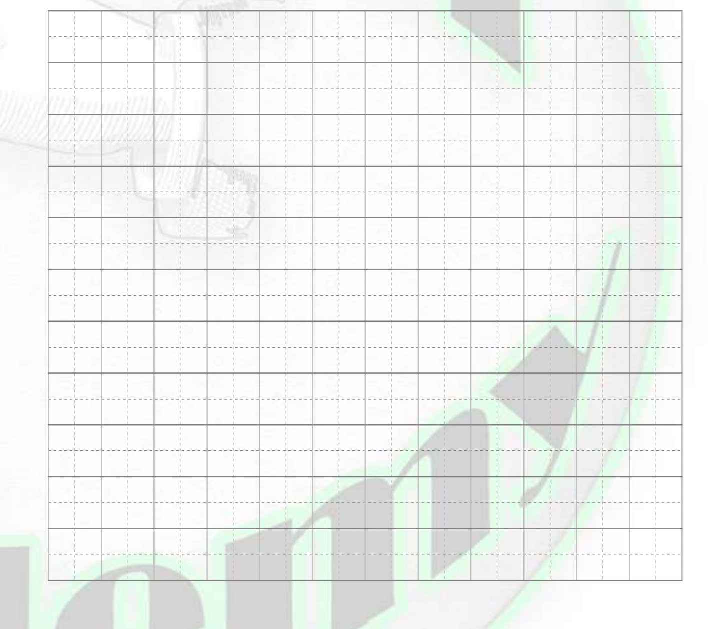
3-2 The ascending and descending cumulative frequency table

1) **The following frequency table shows the weekly wages in pounds of 50 workers in one factory :**

Sets of wages	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (frequency)	5	12	22	7	4	50

The required is forming the ascending cumulative frequency table and representing it graphically then find :

The upper boundaries of sets	Frequency
Less than 54	
Less than 58	
Less than 62	
Less than 66	
Less than 70	
Less than 74	



a. The number of workers whose wages are less than 60 pounds.

....

b. The percentage of the number of workers whose wages are less than 60 pounds.

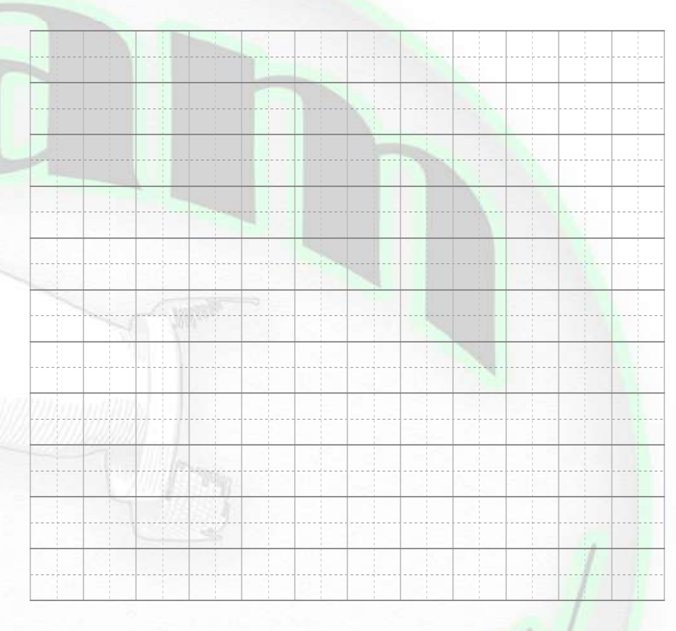
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2) **The following frequency table shows the weekly wages of 50 workers in one factory :**

Sets	54 –	58 –	62 –	66 –	70 –	Total
No. of workers (Frequency)	5	12	22	7	4	50

The required is forming the descending cumulative frequency table and representing it graphically then find :

The lower boundaries of sets	Frequency
54 and more	
58 and more	
62 and more	
66 and more	
70 and more	
74 and more	



a. The number of workers whose weekly wages are 60 pounds or more.

....

b. The percentage of the number of workers whose weekly wages are 60 pounds or more.

....

3) **The following table shows the frequency distribution of marks of 40 students in math exam :**

Sets	10 –	20 –	30 –	40 –	50 –	**Total**
Frequency	4	8	12	10	6	40

Graph each of :

1. The ascending cumulative frequency curve.
2. The descending cumulative frequency curve.

The upper boundaries of sets	**Frequency**

The lower boundaries of sets	**Frequency**

3-3 The Mean

$$\text{The mean of a set of values} = \frac{\text{The total of values}}{\text{Number of values}}$$

1) **The following table shows the distribution of the marks of 50 pupils in mathematics :**

Sets	10 –	20 –	30 –	40 –	50 –	Total
Frequency	8	12	14	9	7	50

Find the mean of these marks.

Set	Centre of the set « x »	Frequency « f »	$x \times f$
10 –		8	
20 –		12	
30 –		14	
40 –		9	
50 –		7	
Total		50	

∴ The mean = $\frac{\dots\dots\dots}{\dots\dots\dots}$ = ...

2) The following table shows the weekly wages in pounds of 50 workers in a factory :

Sets	5 –	15 –	25 –	35 –	45 –	Total
Frequency	7	10	12	13	8	50

Find the mean of the wage of the worker in pounds.

Set	Centre of the set « x »	Frequency « f »	$x \times f$
5 –			
15 –			
25 –			
35 –			
45 –			
Total			

∴ The mean = $\frac{\dots\dots\dots}{\dots\dots\dots}$ = …………

3-4 The Median

The median is the middle value in a set of values after arranging it ascendingly or descendingly.

Find the median

1) 42 , 23 , 17 , 30 and 20

2) 27 , 13 , 23 , 24 , 13 , 21

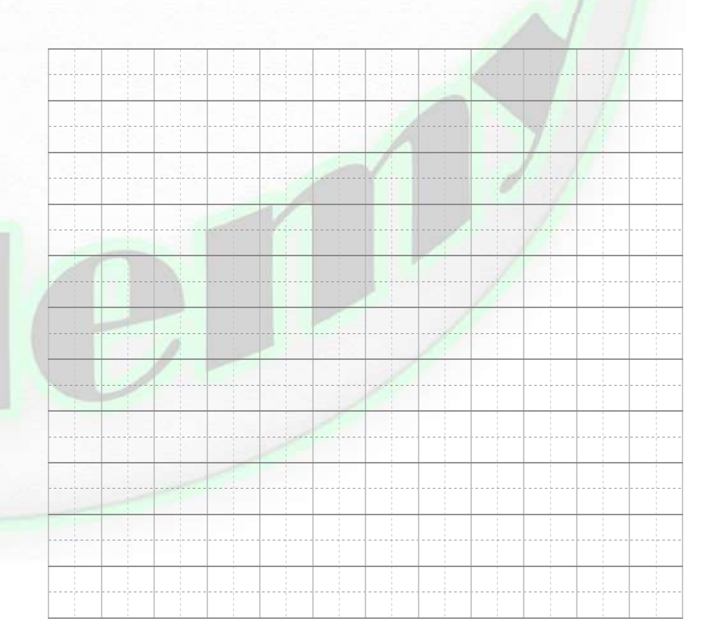
3) **The following table shows the frequency distribution of marks of 50 students in math exam :**

Sets of marks	0 –	10 –	20 –	30 –	40 –	50 –	**Total**
Number of students	2	5	8	19	14	2	**50**

Find the median mark of the student.

Using the ascending cumulative frequency curve :

The upper boundaries of sets	Frequency
Less than 0	
Less than 10	
Less than 20	
Less than 30	
Less than 40	
Less than 50	
Less than 60	

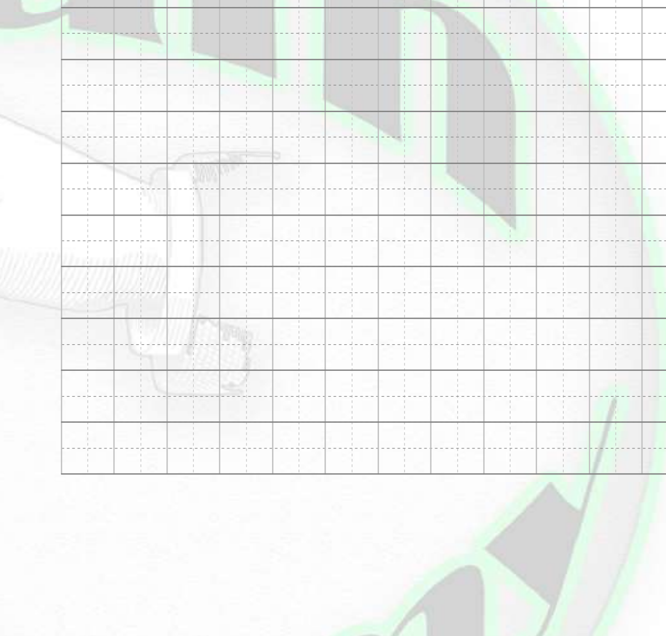


4) **Using the ascending or descending cumulative frequency curve , find the median of the following frequency distribution :**

Sets	4 –	8 –	12 –	16 –	20 –	Total
Frequency	2	4	8	6	4	24

Using the descending cumulative frequency curve :

The lower boundaries of sets	Frequency



3-5 The Mode

The mode of a set of values is the most common value in the set

Find the mode mark

1) 7 , 3 , 4 , 1 , 7 , 9 , 7 , 4

....

2) **The following is the frequency distribution of marks of 100 pupils in one of the exams :**

Set of marks	10 –	20 –	30 –	40 –	50 –	**Total**
Number of pupils	16	24	30	20	10	**100**

Find the mode mark for these pupils.

The mode = ...

3) **Find the mode for the following frequency distribution :**

Sets	2 –	4 –	6 –	8 –	10 –	Total
Frequency	3	10	12	10	5	40

The mode = ……………

Unit 4
Medians of Triangles



Mr. Eslam Youssif
0122 67 666 55

4-1 Revision

The relations between the angles

- **Complementary angles :**

  The two angles are said to be complementary, if the sum of their measures is 90°

- **Supplementary angles :**

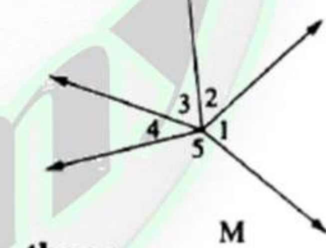
  The two angles are said to be supplementary, if the sum of their measures is 180°

- **Vertically opposite angles (V.O.A.) :**

  If two straight lines intersect , then each two vertically opposite angles are equal in measure.

- **Accumulative angles at a point :**

  The sum of measures of the accumulative angles at a point is 360°

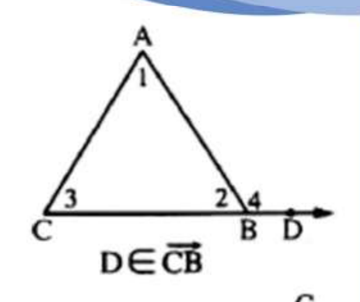


Parallelism

If a straight line intersects two parallel straight lines , then :

- **Each two alternate angles are equal in measure.**
- **Each two corresponding angles are equal in measure.**
- **Each two interior angles in the same side of the transversal are supplementary.**

Triangle



- The sum of measures of the interior angles of a triangle = 180°
- The measure of the exterior angle of a triangle equals the sum of the measures of its non-adjacent interior angles.

Pythagoras' Theorem :

- $(BC)^2 = (AB)^2 + (AC)^2$
- $(AB)^2 = (BC)^2 - (AC)^2$
- $(AC)^2 = (BC)^2 - (AB)^2$

Congruence of two triangles :

The two triangles are congruent if one of the following cases is satisfied :

- Congruence of two sides and the included angle of one triangle to the corresponding parts of the other triangle.

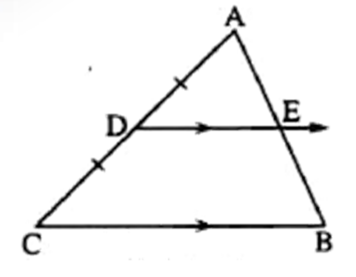


- Congruence of two angles and the side drawn between their vertices of one triangle to the corresponding parts of the other triangle.
- Congruence of each side of one triangle to the corresponding side of the other tirangle.



- Two right-angled triangles are congruent , if the hypotenuse and a side of one triangle are congruent to the corresponding parts of the other triangle.

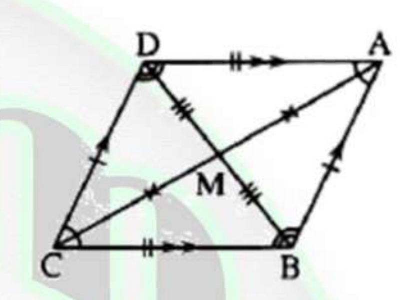




The polygon

- The sum of measures of the interior angles of a polygon with n sides equals $(n - 2) \times 180°$
- The measure of each interior angle in a regular polygon with n sides $= \frac{(n-2) \times 180°}{n}$

The parallelogram and its properties



- each two opposite sides are equal in length and parallel.
- each two opposite angles are equal in measure.
- each two consecutive angles in a parallelogram is 180°
- The two diagonals in a parallelogram bisect each other.

4-2 Medians of triangle

The median of the triangle is the line segment drawn from any vertex of the triangle vertices to the midpoint of the opposite side of this vertex.

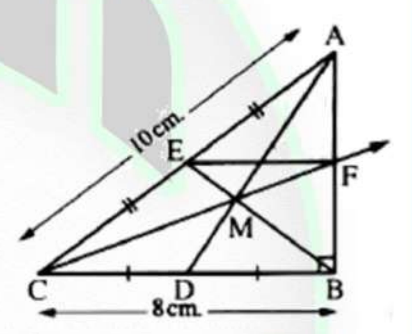
➢ **Theorem (1)**

The medians of a triangle are concurrent.

1) **In the opposite figure :**

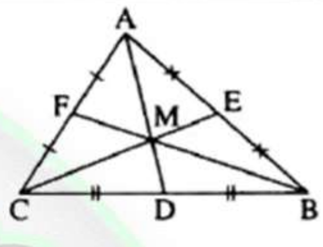
ABC is a right-angled triangle at B in which : AC = 10 cm. , BC = 8 cm. , D and E are the midpoints of $\overline{BC}$ and $\overline{AC}$ respectively where $\overline{AD} \cap \overline{BE} = \{M\}$

Draw $\overrightarrow{CM}$ to cut $\overline{AB}$ at F , Find the perimeter of Δ AFE



Theorem (2)

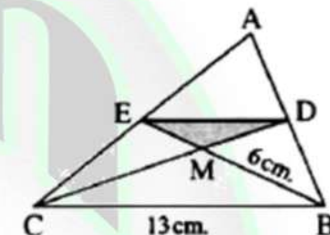
The point of concurrence of the medians of the triangle divides each median in the ratio of 1 : 2 from its base.



2) **In the opposite figure :**

In Δ ABC , $\overline{CD}$ and $\overline{BE}$ are two medians intersecting at M , BM = 6 cm. , BC = 13 cm. and DC = 12 cm.

Find the perimeter of Δ DME



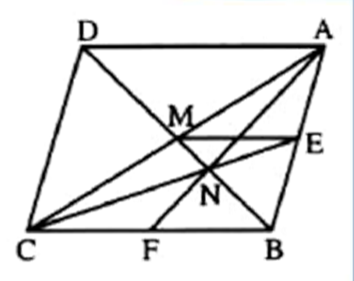
3) **In the opposite figure :**

ABCD is a parallelogram ,

M is the point of intersection of its diagonals ,

$N \in \overline{BM}$ where BN = 2 NM , $\overline{AF} \cap \overline{BD} = \{N\}$ and

$\overrightarrow{CN} \cap \overline{AB} = \{E\}$ **Prove that :** $EM = \frac{1}{2} BC$



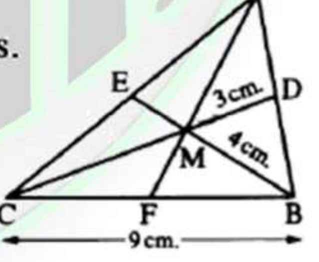
4) **In the opposite figure :**

ABC is a triangle and M is the point of intersection of its medians.

If MD = 3 cm. , BM = 4 cm. and BC = 9 cm. ,

complete the following :

1 BF = ... cm. **2** MC = ... cm. **3** ME = ... cm.



4-3 (Follow) Median of triangle

➢ **Theorem (3)**

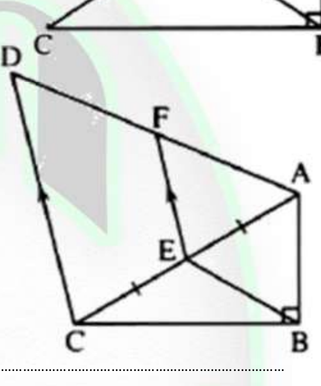
In the right-angled triangle , the length of the median from the vertex of the right angle equals half the length of the hypotenuse.

1) **In the opposite figure :**

ABCD is a quadrilateral in which m (∠ ABC) = 90° , AC = CD ,

E is the midpoint of $\overline{AC}$ and F ∈ $\overline{AD}$ such that $\overline{EF}$ // $\overline{CD}$

Prove that : BE = EF



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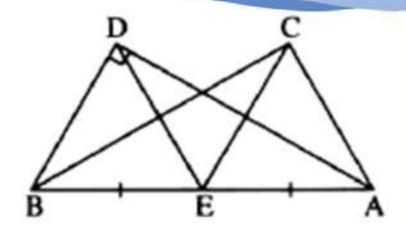
....

➢ **The converse of theorem (3)**

If the length of the median drawn from a vertex of a triangle equals half the length of the opposite side to this vertex, then the angle at this vertex is right.

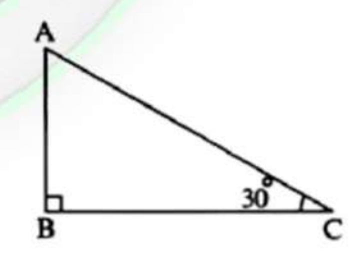
2) **In the opposite figure :**

ABD is a right-angled triangle at D , E is the midpoint of $\overline{AB}$ and CE = DE **Prove that :** m (∠ ACB) = 90°

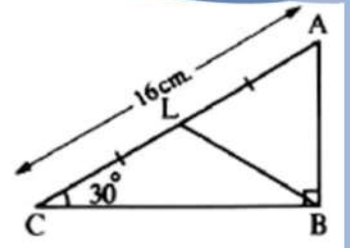


➢ **Corollary**

The length of the side opposite to the angle of measure 30° in the right-angled triangle equals half the length of the hypotenuse.



3) **In the opposite figure :**

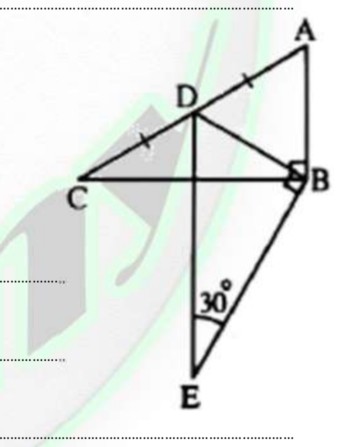


ABC is a triangle in which m (∠ ABC) = 90° ,

m (∠ C) = 30° and AC = 16 cm. , L is the midpoint of $\overline{AC}$

Find : the length of each of $\overline{AB}$ and $\overline{BL}$ and the perimeter of Δ ABL

4) **In the opposite figure :**



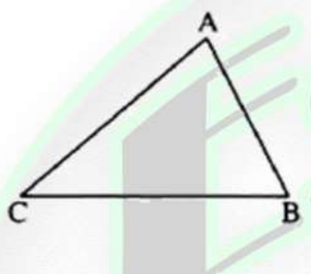
m (∠ ABC) = m (∠ DBE) = 90° , D is the midpoint of $\overline{AC}$

and m (∠ E) = 30° prove that : AC = DE

4-4 The isosceles triangle

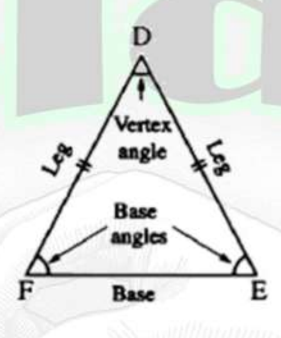
Triangles are classified according to the lengths of their sides into three types which are :

- **Scalene triangle.**



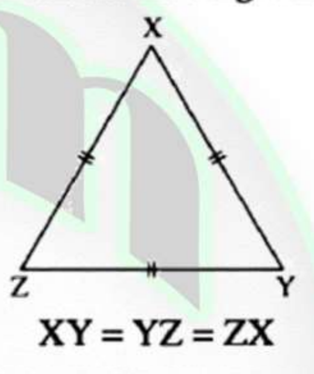
AB ≠ BC ≠ CA

- **Isosceles triangle.**
  (two sides are congruent).



DE = DF

- **Equilateral triangle.**
  (three sides are congruent).



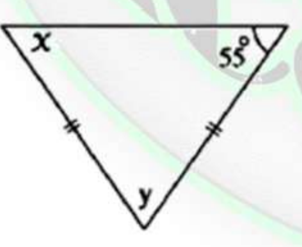
XY = YZ = ZX

- **Theorem (1)**

  The base angles of the isosceles triangle are congruent.

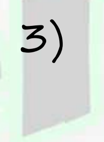
In each of the following figures, find the values of the symbols used as a measure for the angle :

1)



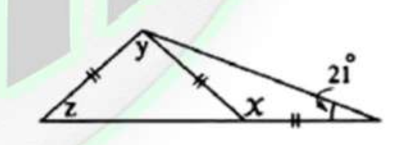
x = ...° , y = ...°

2)



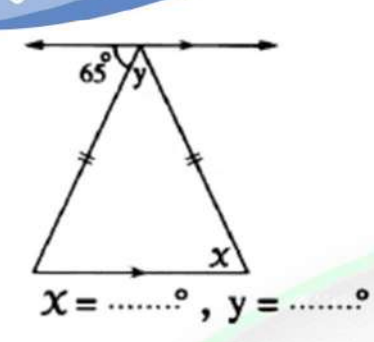
x = ...° , y = ...°

3)



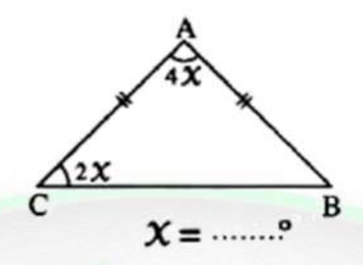
x = ...° , y = ...° , z = ...°

4)



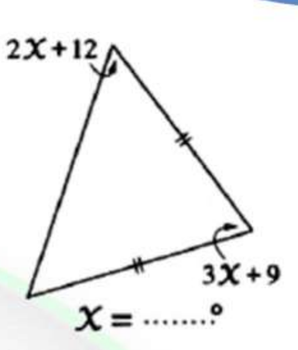
$x =$ ....° , $y =$ ....°

5)



$x =$ ....°

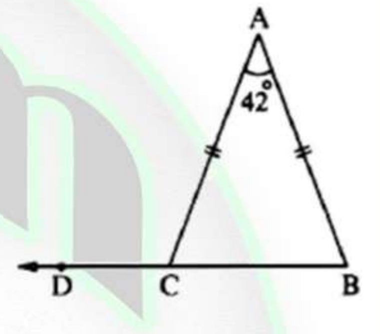
6)



$x =$ ....°

7) **In the opposite figure :**

AB = AC , m (∠ A) = 42° and D ∈ $\overrightarrow{BC}$ **Find :** m (∠ ACD)



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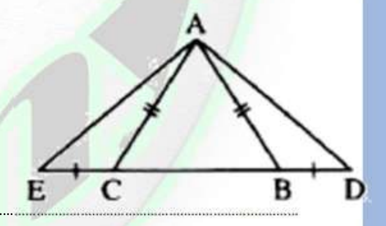
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8) **In the opposite figure :**

B ∈ $\overline{DE}$, C ∈ $\overline{DE}$, AB = AC and BD = CE **Prove that :** AD = AE



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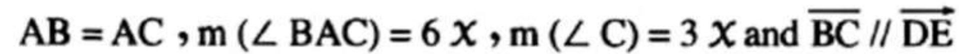
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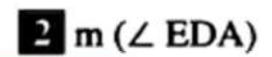
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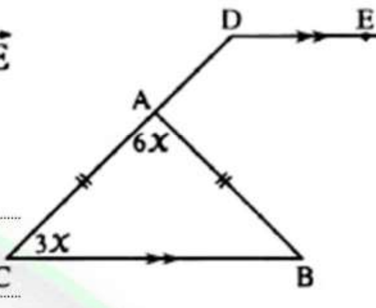
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9) **In the opposite figure :**


AB = AC , m (∠ BAC) = 6 x , m (∠ C) = 3 x and $\overline{BC}$ // $\overrightarrow{DE}$

Find : **1** The value of x  **2** m (∠ EDA)

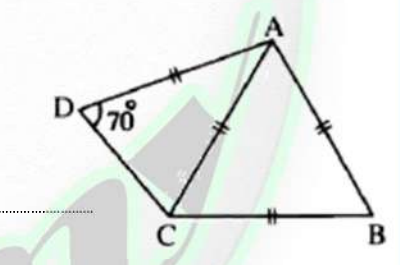


➢ **Corollary**

If the triangle is equilateral , then it is equiangular where each angle measure is 60°

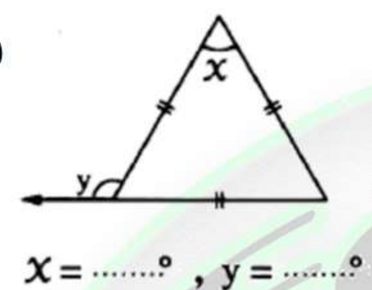
10) **In the opposite figure :** AB = BC = CA = AD

m (∠ D) = 70° **Find :** **1** m (∠ BCD) **2** m (∠ BAD)



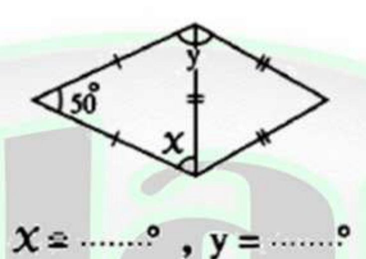
In each of the following figures , find the values of the symbols used as a measure for the angle :

11)



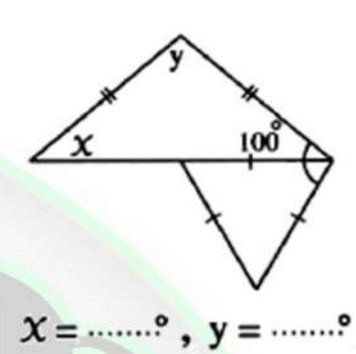
$x =$ ……..° , $y =$ ……..°

12)



$x =$ ……..° , $y =$ ……..°

13)



$x =$ ……..° , $y =$ ……..°

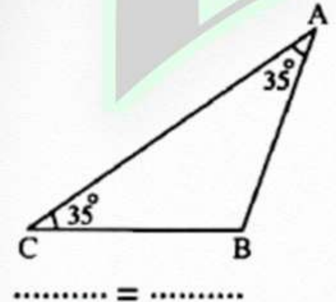
4-5 The converse of the isosceles triangle theorem

➢ **Theorem (2)**

If two angles of a triangle are congruent, then the two sides opposite to these two angles are congruent and the triangle is isosceles.

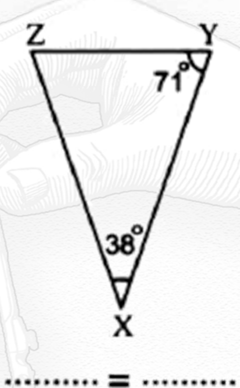
In each of the following figures , write the equal sides in length:

1)



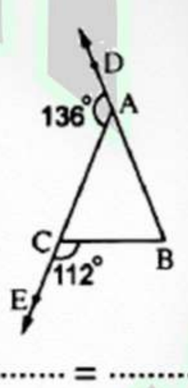
.... = ...

2)



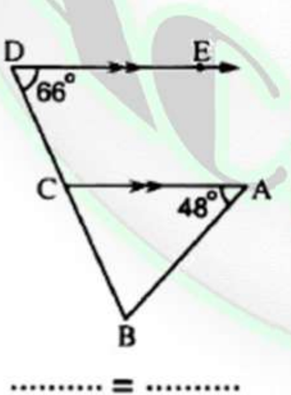
.... = ...

3)



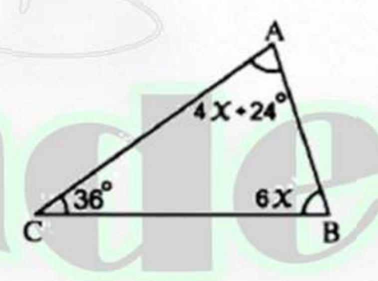
.... = ...

4)



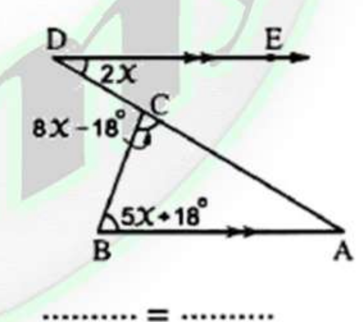
.... = ...

5)



.... = ...

6)



.... = ...

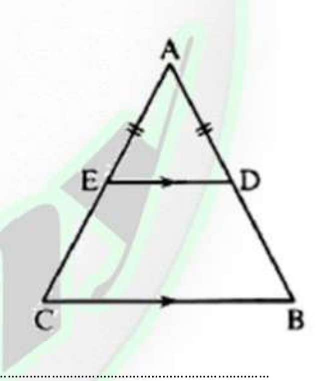
7) ABC is a triangle in which m (∠ A) = 2 m (∠ B) = 72°

Prove that : Δ ABC is an isosceles triangle.



8) **In the opposite figure :** $D \in \overline{AB}$ and $E \in \overline{AC}$

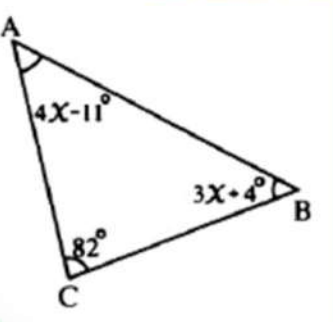
where AD = AE and $\overline{DE}$ // $\overline{BC}$ **Prove that :** DB = EC



9) **In the opposite figure :**

If m (∠ A) = $4x - 11°$, m (∠ B) = $3x + 4°$, m (∠ C) = 82°

Prove that : Δ ABC is an isosceles triangle.



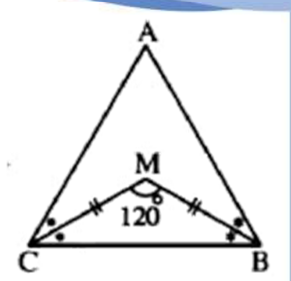
➢ **Corollary**

If the angles of a triangle are congruent, then the triangle is equilateral.

10) **In the opposite figure :** $\overrightarrow{BM}$ bisects ∠ B , $\overrightarrow{CM}$ bisects ∠ C ,

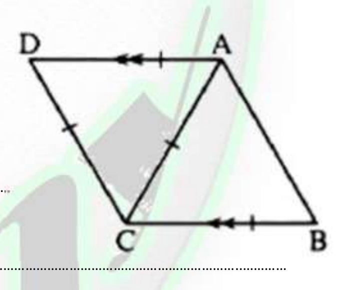
MB = MC and m (∠ BMC) = 120°

Prove that : Δ ABC is an equilateral triangle.



11) **In the opposite figure :** AD = DC = CB = CA , $\overline{AD}$ // $\overline{BC}$

prove that : Δ ABC is an equilateral triangle :



4-6 Corollaries of isosceles triangle theorems

- **Corollary (1)**

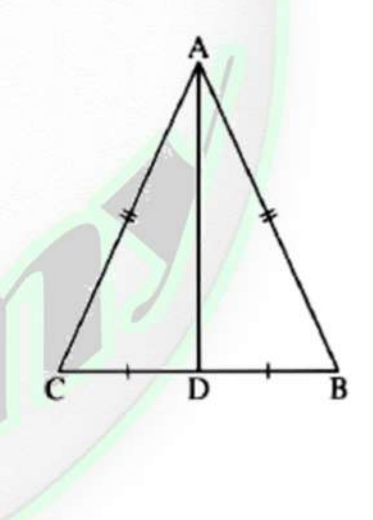
The median of an isosceles triangle from the vertex angle bisects it and is perpendicular to the base.

- **Corollary (2)**

The bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to it.

- **Corollary (3)**

The straight line drawn passing through the vertex angle of an isosceles triangle perpendicular to the base bisects each of the base and the vertex angle.



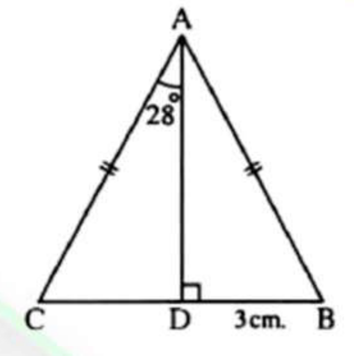
1) **In the opposite figure :**

ABC is an isosceles triangle where

AB = AC and D $\in \overline{BC}$ such that $\overline{AD} \perp \overline{BC}$,

m (∠ CAD) = 28° and BD = 3 cm. Find :

1 m (∠ BAC) **2** the length of $\overline{BC}$



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➢ **Axis of symmetry of a line segment**

Definition

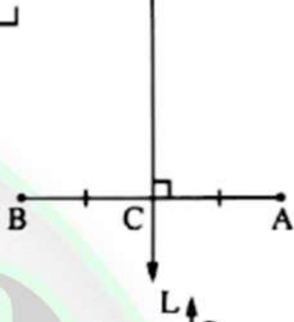
The straight line perpendicular to a line segment at its middle is called the axis of symmetry for that line segment , in brief it is known as the axis of a line segment.

➢ **Property**

Any point on the axis of symmetry of a line segment is at equal distances from its terminals (end points).

➤ **The converse of the previous property is true**

i.e. If a point is at equal distances from the two terminals of a line segment , then this point lies on the axis of this line segment.



2) **In the opposite figure :**

The straight line L is the axis of $\overline{AB}$

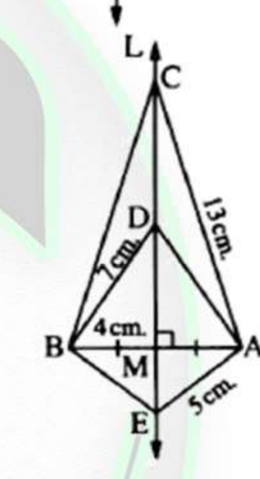
If the points C , D and E belong to the straight line L , $L \cap \overline{AB} = \{M\}$

where AC = 13 cm. ,

DB = 7 cm. , AE = 5 cm. and

MB = 4 cm. Find the length of each of

$\overline{CB}$, $\overline{DA}$, $\overline{EB}$ and $\overline{MA}$



3) ΔABC is an isosceles triangle where $AB = AC$, $\overrightarrow{BX}$ bisects $\angle ABC$ and intersects $\overline{AC}$ at X , $\overrightarrow{CY}$ bisects $\angle ACB$ and intersects $\overline{AB}$ at Y If $\overrightarrow{BX} \cap \overrightarrow{CY} = \{M\}$,

prove that : $\overleftrightarrow{AM} \perp \overline{BC}$

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➤ **Axis of symmetry of the isosceles triangle**

The isosceles triangle has one axis of symmetry.

It is the straight line drawn from the vertex angle perpendicular to its base.

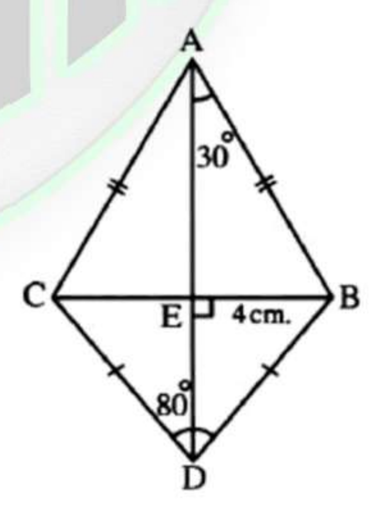
4) **In the opposite figure :**

ABDC is a quadrilateral in which :

$AB = AC$, $BD = CD$, $\overline{AD} \perp \overline{BC}$,

$\overline{AD} \cap \overline{BC} = \{E\}$, $m(\angle BAD) = 30°$,

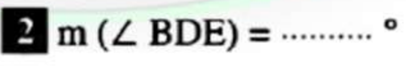
$m(\angle BDC) = 80°$ and $BE = 4$ cm.



Complete the following :

1 $m(\angle DAC) =$ .... °

2 $m(\angle BDE) =$ .... °



3 $m(\angle ACB) =$ ....°

4 $EC =$ .... cm.

5 $AC =$ .... cm.

Unit 5

Inequality



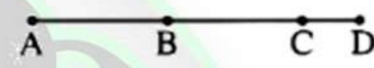
5-1 Inequality

➤ **Axioms of inequality relation**

For any four numbers a , b , c and d

1. If $a > b$, then $a + c > b + c$
2. If $a > b$, then $a - c > b - c$
3. If $a > b$, $c > 0$, then $a c > b c$
4. If $a > b$, $b > c$, then $a > c$
5. If $a > b$, $c > d$, then $a + c > b + d$

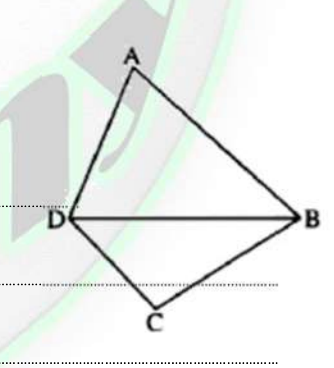
1) **In the opposite figure :**



If B and C belong to $\overline{AD}$ such that $AB > CD$ **Prove that :** $AC > BD$

2) **In the opposite figure :** If $m(\angle ADB) > m(\angle ABD)$,

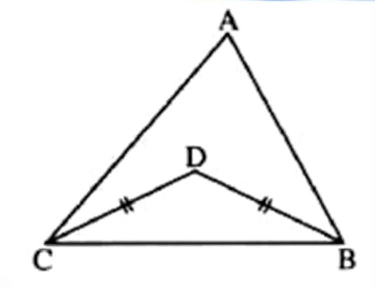
$m(\angle CBD) < m(\angle CDB)$ **Prove that :** $m(\angle ADC) > m(\angle ABC)$



3) **In the opposite figure :**

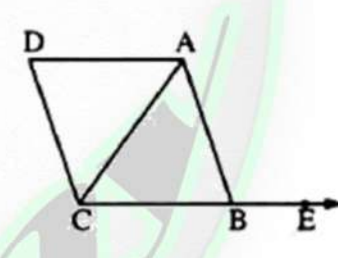
If m (∠ ABC) > m (∠ ACB) and BD = DC

Prove that : m (∠ ABD) > m (∠ ACD)



4) **In the opposite figure :** ABCD is a parallelogram

$E \in \overrightarrow{CB}$ **prove that :** m (∠ ABE) > m (∠ ACD)

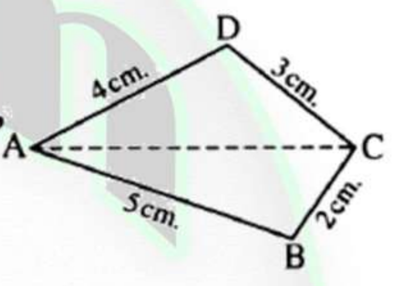


5-2 Comparing the measures of angles in a triangle

➢ **Theorem**

In a triangle , if two sides have unequal lengths , the longer is opposite to the angle of the greater measure.

1) **In the opposite figure :**



ABCD is a quadrilateral in which AB = 5 cm. , BC = 2 cm. , CD = 3 cm. and DA = 4 cm.

Prove that : m (∠ DCB) > m (∠ DAB)

2) ABC is a triangle in which AB > AC and ∠ BAC is bisected by $\overrightarrow{AD}$ which intersects $\overline{BC}$ at D

Prove that : ΔABD is an obtuse-angled triangle.

A
C D B

3) **In the opposite figure :**

ABC is a triangle in which AB > AC , D and E are the midpoints of $\overline{AB}$ and $\overline{AC}$ respectively **Prove that :** m (∠ AED) > m (∠ ADE)

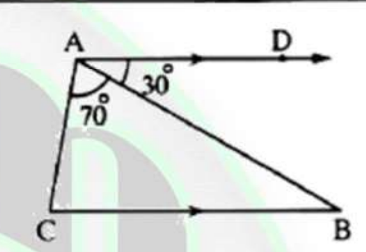
A
E D
C B

5-3 Comparing the lengths of sides in a triangle

➢ **Theorem**

In a triangle, if two angles are unequal in measure, then the greater angle in measure is opposite to a side greater in length than that opposite to the other angle.

1) **In the opposite figure :**



ABC is a triangle in which m (∠ BAC) = 70°,

$\overrightarrow{AD}$ // $\overline{BC}$ and m (∠ DAB) = 30°

Prove that : AB > AC

....

....

....

....

....

....

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....

➢ **Corollary (1)**

In the right-angled triangle, the hypotenuse is the longest side.

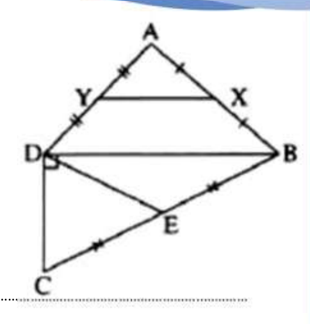
➢ **Corollary (2)**

The length of the perpendicular line segment drawn from a point outside a straight line to this line is shorter than any line segment drawn from this point to the given straight line.

2) **In the opposite figure :**

ABCD is a quadrilateral. X , Y and E are the midpoints of $\overline{AB}$, $\overline{AD}$ and $\overline{BC}$ respectively and m (∠ BDC) = 90°

Prove that : DE > XY

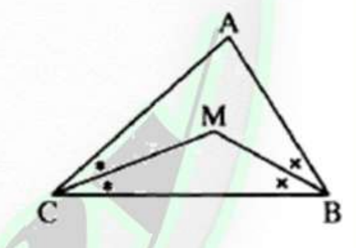


3) **In the opposite figure :**

ABC is a triangle in which AC > AB ,

$\overrightarrow{BM}$ bisects ∠ ABC and $\overrightarrow{CM}$ bisects ∠ ACB

prove that : MC > MB



5-4 Triangle inequality

In any triangle, the sum of the lengths of any two sides is greater than the length of the third side.

Show which of the following numbers can be side lengths of a triangle.

1) 5 , 7 , 12 ....

2) 14 , 9 , 7 ....

3) 12 cm. , 5 cm. , 7.5 cm. ....

4) 4 , 6 , 11 ....

5) 8 , 18 , 8 ....

6) 2 cm. , 3 cm. , 4 cm. ....

Find the interval to which the length of the third side of each of the following triangles belongs if the two lengths of the other two sides are :

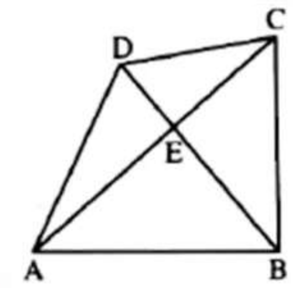
7) 4 cm. , 3 cm.
....
....

8) 4.5 cm. , 7.5 cm.
....
....

9) $2\sqrt{5}$ cm. , $2\sqrt{5}$ cm.
....
....

10) 6 cm. , 5 cm.
....
....

11) **In the opposite figure :**



ABCD is a quadrilateral , whose diagonals intersect at E

Prove that : $AC + BD > BC + AD$

